This is a trial paper and does not necessarily reflect the content or format of the final HSC paper

SCECGS Redlands

Trial Higher School Certificate 2000

3/4 Unit Mathematics

Total Time Allowed: 2 hours
(plus 5 minutes reading time)

Instructions to Candidates:

- There are seven questions each worth ~~15~~ 12 marks.
- Each question attempted is to be returned in a separate writing booklet clearly marked *Question 1*, *Question 2 etc* on the cover. Each booklet *must* show your student number.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Standard Integrals are printed at the end of the paper.
- Even if you have not attempted a question submit a numbered blank booklet clearly showing your student number.
- In every question all necessary working is to be shown *in pen* except for *diagrams* which should be large and drawn *in pencil*.
- NSW Board of Studies approved calculators may be used.

Question 1 (15marks) *Use a separate writing booklet* **Marks**

(a) Find (i) $\int \frac{x+3}{x^2+6x-7}dx$ **5**

(ii) $\int_0^{\frac{\pi}{4}} \cos^2 x\,dx$

(b) By using the substitution $u = 1+x^2$ or otherwise find **4**

$$\int \frac{4x}{\sqrt{1+x^2}}dx$$

(c) The velocity of a particle is given by $\frac{dx}{dt}=4t-7$ **2**

Given that $x=3$ when $t=0$, find an expression for the distance x at any time t.

(d) Use the substitution $x = 3\sin\theta$ to solve $\int_0^3 x\sqrt{9-x^2}dx$ **4**

Question 2 (15 marks) *Use a separate writing booklet* **Marks**

(a) Using exact values, show that $\sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$ **3**

(b) Find the general solution for x given **4**

$\cos^2 x - 3\sin x + 3 = 0$

(c) Prove the identity **4**

$$\frac{\cos A+\sin A}{\cos A-\sin A}-\frac{\cos A-\sin A}{\cos A+\sin A} = 2\tan 2A$$

(d) $x = 2$ is a zero of the polynomial $P(x) = x^3 + x^2 + kx - 4$ **4**

(i) Determine the value of k

(ii) Determine all the factors of $P(x)$

Question3 (15 marks) *Use a separate writing booklet* **Marks**

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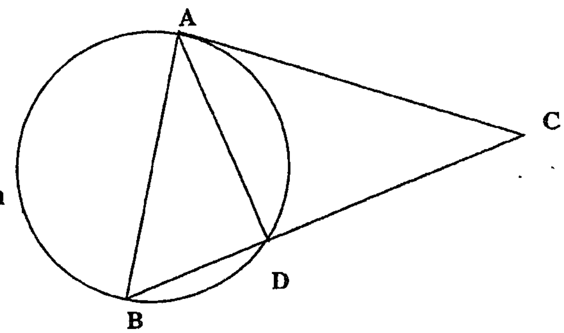
(a) In the diagram, AC is a tangent to the circle at A and **angle CAB** = $90°$



(i) Show that Δ **ABD** ||| Δ **CAD**

(ii) given that **BD=4cm** and **CD=6cm** calculate the length of **AD**

(iii) Calculate the **radius** of the circle.

(b) A pump is used to inflate a spherical balloon. It is found that when the radius of the balloon is increasing at the rate of **1cm/s**, the radius is **40cm.** **7**

(i) At what rate is the **volume** of the balloon increasing when the radius is **40cm?**

(ii) Determine the rate at which the **surface area** of the balloon would increase when its radius is **40cm.**

Question 4 (15 marks) *Use a separate writing booklet* **Marks**

(a) Differentiate w.r.t. x **3**

$$y = \cos^{-1}(\sin x)$$

(b) Write down the domain and range of the function **3**

$$y = 2\cos^{-1} 3x$$

Draw a neat sketch of the function.

(c) Show that $\displaystyle\int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{9+4x^2} = \frac{\pi}{36}$ **4**

(d) (i) Show that $\cos^{-1}(-x) = \pi - \cos^{-1} x$ **5**

(ii) Hence evaluate $\sin^{-1}\frac{1}{2} + \cos^{-1}(-\frac{\sqrt{3}}{2})$

Question5 (15 marks) *Use a separate writing booklet* **Marks**

(a) A particle moves in a straight line so that its position x from a fixed point O at time t is given by: **10**

$x = 3\sin 2t + 4\cos 2t$

(i) If the motion is expressed in the form $x = r\sin(2t+\alpha)$, evaluate the constants r and α.

(ii) Show that the motion is Simple Harmonic.

(iii) What is the period of oscillation?

(iv) Determine the maximum displacement from the centre of motion.

(b) Prove by mathematical induction that **5**

$$1 + 4 + 7 + \ldots\ldots + (3n-2) = \frac{n(3n-1)}{2}$$

Question6 (15 marks) *Use a separate writing booklet* **Marks**

(a) Newton's law of cooling can be represented mathematically as **7**
$\frac{dT}{dt} = -k(T - T_0)$ where $\frac{dT}{dt}$ is the rate of cooling, k is a constant, T is the temperature at any instant and T_0 is the room temperature.

(i) Show that $T = T_0 + Ae^{-kt}$ is a solution to the above equation.

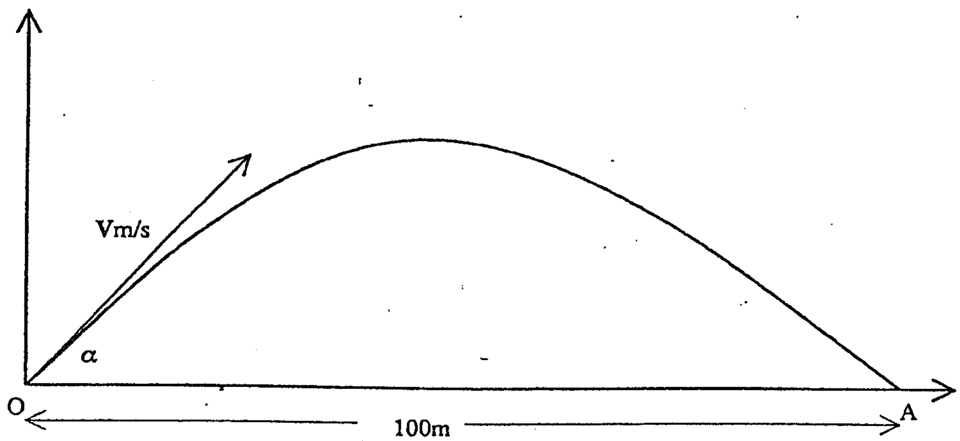
(ii) A cup of tea cools from 85°C to 80°C in 1minute. Taking room temperature as 25°C, find, to the nearest degree, the temperature of the tea after 4 minutes.

(b) Determine the co-ordinates of the point P that divides the line joining A(-1,6) and B(4,-6) externally in the ratio 2:3 **4**

(c) Find the acute angle between the lines $2x - y - 3 = 0$ and $x - 3y - 7 = 0$ **4**

Question7 (15 marks) *Use a separate writing booklet* **Marks**

(a) A projectile is fired from ground level at an angle α to the horizontal, with initial velocity V metres per second. The projectile returns to the ground after 5seconds, 100m away from the point of projecion.. Assume that the acceleration due to gravity is $10ms^{-2}$, and that the ground is horizontal. 12 [(a) + (b)]



(i) Beginning with $\ddot{x} = 0$ and $\ddot{y} = -10$, derive equations for velocity and displacement. (i.e. for $\dot{x}, \dot{y}, x, y$) as functions of time.

(ii) Calculate the angle of projection to the nearest minute

(iii) Find the initial velocity in exact form.

(iv) Find the maximum height attained by the projectile.(to the nearest metre)

(b) At the same time that the projectile in (a) is fired, a man, 2metres tall, (unaware that the projectile has been fired) walks from A towards O. A few seconds later, he is hit on the top of his head by the projectile. Show that this accident occurs at a distance of approximately 98 metres from O.

(c) The function $y = 4x^2 - 11x + 7$ has an approximate root at 0.73 3
Using one application of Newton's method, determine a closer approximation to the root.

2000.

(i) $\int \frac{x+3}{x^2+6x-7}\,dx$

$= \frac{1}{2}\log_e(x^2+6x-7) + C$

(ii) $\int_0^{\pi/4} \cos^2 x\,dx$

$\cos 2x = 2\cos^2 x - 1$

$\therefore 2\cos^2 x = 1+\cos 2x$

$\therefore \cos^2 x = \frac{1+\cos 2x}{2}$

$= \int_0^{\pi/4} \frac{1+\cos 2x}{2}\,dx$

$= \left[\frac{x}{2} + \frac{\sin 2x}{4}\right]_0^{\pi/4}$

$= \left[\left(\frac{\pi}{8} + \frac{\sin 2\times\frac{\pi}{4}}{4}\right) - (0)\right]$

$= \frac{\pi}{8} + \frac{\sin\frac{\pi}{2}}{4}$

$= \frac{\pi}{8} + \frac{1}{4}$

$= \frac{1}{8}(\pi+2)$

(b). $\int \frac{4x}{\sqrt{1+x^2}}\,dx$

$u = 1+x^2$

$\therefore du = 2x\,dx$

$\therefore dx = \frac{1}{2x}\,du$

$= \int \frac{4x}{u^{1/2}} \cdot \frac{1}{2x}\,du$

$= 2\int u^{-1/2}\,du$

$= 2 \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + A$

$= 4\sqrt{u} + A$

$= 4\sqrt{1+x^2} + A$

(c). $\frac{dx}{dt} = 4t-7$

$\therefore \int dx = \int (4t-7)\,dt + A$

$\therefore x = 2t^2 - 7t + A$

When $t=0$, $x=3$

$\therefore 3 = A$

$\therefore \boxed{x = 2t^2 - 7t + 3}$

$$\int_0^{3} x\sqrt{9-x^2}\,dx \quad , \quad x = 3\sin\theta$$

$\therefore dx = 3\cos\theta\, d\theta$.

When $x=3$, $\sin\theta = 1$, $\therefore \theta = \frac{\pi}{2}$

When $x=0$, $\sin\theta=0$, $\therefore \theta = 0$

$$= \int_0^{\pi/2} 3\sin\theta\sqrt{9-9\sin^2\theta}\cdot 3\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} 3\sin\theta\cdot 3\cos\theta\cdot 3\cos\theta\, d\theta$$

$$= 27\int_0^{\pi/2} \cos^2\theta\cdot\sin\theta\, d\theta$$

$$= 27\cdot\left[\frac{\cos^3\theta}{3}\right]_0^{\pi/2}$$

$$= 9[0-1]$$

$$= -9$$

(a) To Show: $\sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$$\sin 75^\circ = \sin(45^\circ + 30^\circ)$$

$$= \sin 45^\circ\cdot\cos 30^\circ + \cos 45^\circ\cdot\sin 30^\circ$$

$$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\times\frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

b. $\cos^2 x - 3\sin x + 3 = 0$

ie $1-\sin^2 x - 3\sin x + 3 = 0$

ie $-\sin^2 x - 3\sin x + 4 = 0$

ie $\sin^2 x + 3\sin x - 4 = 0$

ie $(\sin x + 4)(\sin x - 1) = 0$

$\therefore \sin x = 1, \cancel{-4}$

$\therefore \quad x = n\pi + (-1)^n \frac{\pi}{2}$

$$\frac{\cos A+\sin A}{\cos A-\sin A}-\frac{\cos A-\sin A}{\cos A+\sin A}=2\tan 2A$$

$$LHS=\frac{\cos A+\sin A}{\cos A-\sin A}-\frac{\cos A-\sin A}{\cos A+\sin A}$$

$$=\frac{(\cos A+\sin A)^2-(\cos A-\sin A)^2}{\cos^2 A-\sin^2 A}$$

$$=\frac{\cos^2 A+2\sin A\cos A+\sin^2 A-[\cos^2 A-2\sin A\cos A+\sin^2 A]}{\cos 2A}$$

$$=\frac{\cancel{\cos^2 A}+2\sin A\cos A+\cancel{\sin^2 A}-\cancel{\cos^2 A}+2\sin A\cos A-\cancel{\sin^2 A}}{\cos 2A}$$

$$=\frac{4\sin A\cos A}{\cos 2A}$$

$$=\frac{2\sin 2A}{\cos 2A}$$

$$=2\tan 2A$$

$$=RHS$$

(d)

$$P(x)=x^3+x^2+kx-4$$

$x=2$ is a solution

$\therefore P(2)=0$

$$8+4+2k-4=0$$

$$2k=-8$$

$$k=-4$$

$$P(x)=x^3+x^2-4x-4$$

$$\begin{array}{r}
x^2+3x+2 \\
x-2\,\overline{)\,x^3+x^2-4x-4} \\
x^3-2x^2 \\
\hline
3x^2-4x \\
3x^2-6x \\
\hline
2x-4 \\
2x-4 \\
\hline
\end{array}$$

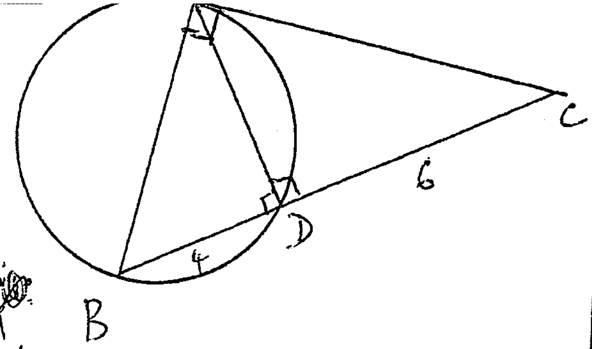
$$x^2+3x+2=(x+2)(x+1)$$

$$\therefore P(x)=(x-2)(x+2)(x+1)$$

Data: AC is a tangent at A

$\angle BAC = 90°$

$BD = 4$ cm, $DC = 6$ cm.



PROVE: (i) $\triangle ABD \parallel \triangle CAD$

(ii) Calculate length of radius

PROOF:

$\angle BAC = 90°$ —— data

$\therefore$ AB is the diameter

$\therefore \angle ADB = 90°$ —— (angle in a semicircle)

In $\triangle^s$ ABD, CAD

$\angle ABD = \angle CAD$ —— $\angle$s in the alternate segment

$\angle ADB = \angle ADC = 90°$ —— (adj $\angle$s on str line)

$\therefore \triangle ABD \parallel \triangle CAD$ (A, A, $\therefore$A)

$AC^2 = CB \cdot CD$ (tangent^2 = product of intercepts of chord)

$= 10 \times 6$

$\therefore AC = \sqrt{60}$

$AD^2 = AC^2 - CD^2$

$= 60 - 36$

$= 24$

$\therefore AD = 2\sqrt{6}$ cm

$AB^2 = AD^2 + BD^2$

$= 24 + 16$

$= 40$

$\therefore AB = 2\sqrt{10}$

$\therefore$ radius $= \frac{1}{2} AB = \sqrt{10}$ cm

5

(b) Given: $\frac{dr}{dt} = 1$ cm/s when $r = 40$ cm.

(i)

$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$ $\qquad V = \frac{4}{3}\pi r^3$

$\therefore \frac{dV}{dr} = \frac{4}{3} \times \pi \times 3r^2$

$= 4\pi r^2$

$\therefore \frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$\therefore$ when $r = 40$ cm.

$\left(\frac{dV}{dt}\right)_{r=40} = 4 \times \pi \times 40^2 \times 1$

$= 6400\pi$ cm^3/s

$3\frac{1}{2}$

11) $S = 4\pi r^2$ ✓ $\therefore \frac{dS}{dr} = 8\pi r$ ✓

$$\frac{dS}{dt} = \frac{dS}{dr} \cdot \frac{dr}{dt}$$ ✓

$$= 8\pi r \cdot \frac{dr}{dt}$$

$$\therefore \left(\frac{dS}{dt}\right)_{r=40cm} = 8 \times \pi \times 40 \times 1$$

$$= \underline{320\pi \ cm^2/s}$$ ✓

Question No 4

(a) $y = \cos^{-1}(\sin x)$

$\therefore \cos y = \sin x$

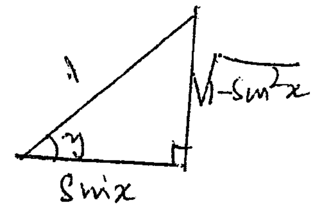
$\therefore -\sin y \frac{dy}{dx} = \cos x$

$$\therefore \frac{dy}{dx} = -\frac{\cos x}{\sin y}$$

$$= -\frac{\cos x}{\sqrt{1-\sin^2 x}}$$

$$= \frac{-\cos x}{\cos x}$$

$$= -1$$



or: $\cos y = \sin x$

$\therefore x + y = 90^\circ$

$\therefore 1 + \frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx} = -1$

(b) $y = 2\cos^{-1} 3x$

$\therefore \frac{y}{2} = \cos^{-1} 3x$

D: $-1 \leq 3x \leq 1$

ie $\underline{-\frac{1}{3} \leq x \leq \frac{1}{3}}$

R: $0 \leq \frac{y}{2} \leq \pi$

$\therefore \underline{0 \leq y \leq 2\pi}$

$$\int_0 \frac{}{9+4x^2}$$

$$= \int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{4\left(\frac{9}{4}+x^2\right)}$$

$$= \frac{1}{4}\int_0^{\sqrt{3}/2} \frac{dx}{\left(\frac{3}{2}\right)^2+x^2}$$

$$= \frac{1}{4}\cdot\frac{1}{\frac{3}{2}}\left[\tan^{-1}\frac{x}{\frac{3}{2}}\right]_0^{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{6}\left[\tan^{-1}\frac{2x}{3}\right]_0^{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{6}\tan^{-1}\frac{1}{\sqrt{3}}]$$

$$= \frac{1}{6}\times\frac{\pi}{6}$$

$$= \frac{\pi}{36}$$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

Let $y = \cos^{-1}(-x)$

$\therefore \cos y = -x$

$\therefore x = -\cos y$

ie $x = \cos(\pi - y)$

$\therefore \cos^{-1}x = \pi - y$

$\therefore y = \pi - \cos^{-1}x$

ie $\cos^{-1}(-x) = \pi - \cos^{-1}x$

(ii) $\sin^{-1}\frac{1}{2} + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= \sin^{-1}\frac{1}{2} + \pi - \cos^{-1}\frac{\sqrt{3}}{2}$

$= \frac{\pi}{6} + \pi - \frac{\pi}{6}$

$= \pi$

(a) $x = 3\sin 2t + 4\cos 2t$

$= 5\left[\frac{3}{\sqrt{3^2+4^2}}\cdot\sin 2t + \frac{4}{\sqrt{3^2+4^2}}\cos 2t\right]$

ie $x = 5\left[\frac{3}{5}\sin 2t + \frac{4}{5}\cos 2t\right]$

$= 5\left[\sin 2t\cos\alpha + \cos 2t\sin\alpha\right]$

when $\cos\alpha = \frac{3}{5}$

ie $x = 5\sin(2t+\alpha)$.

$\dot{x} = 10\cos(2t+\alpha)$

$\ddot{x} = -20\sin(2t+\alpha)$

$= -4\times 5\sin(2t+\alpha)$

$\therefore \ddot{x} = -4x$

This is of the form $\ddot{x} = -n^2x$

$\therefore$ The motion is Simple Harmonic.

Period $= \frac{2\pi}{n} = \frac{2\pi}{2} = \pi$ seconds.

Max. displacement = Amplitude = 5 cm

b) To show that.

$$1+4+7+\cdots+(3n-2) = \frac{n(3n-1)}{2}$$

Let this be true for $n = K$. when $K \geq 1$

$$\therefore 1+4+7+\cdots+(3k-2) = \frac{k(3k-1)}{2} \quad \text{——(i)}$$

Adding the next, $(k+1)^{th}$ term:

$$1+4+7+\cdots+(3k-2)+(3k+1) = \frac{k(3k-1)}{2}+(3k+1)$$

$$= \frac{k(3k-1)+2(3k+1)}{2}$$

$$= \frac{3k^2-k+6k+2}{2}$$

$$= \frac{3k^2+5k+2}{2}$$

$$= \frac{(k+1)(3k+2)}{2} \quad \text{——(2)}$$

This is of the same form as (1) when k is replaced by $k+1$.

$\therefore$ If Statement (1) is true for $n=k$, it is true for $n=k+1$ also.

When $k=1$, LHS in (1) $=1$, RHS in (1) $= \frac{1(3-1)}{2} = 1$.

$\therefore$ By the Principle of Induction: $1+4+7+\cdots+(3n-2) = \frac{n(3n-1)}{2}$

Question No 6 (1)

(a) $$\frac{dT}{dt} = -k(T-T_0)$$

$$\therefore \frac{dT}{T-T_0} = -k\,dt$$

$$\therefore \int \frac{dT}{T-T_0} = -k\int dt + C$$

$$\therefore \log_e(T-T_0) = -kt+C$$

$$\therefore T-T_0 = e^{-kt+c}$$

$$\therefore \boxed{T = T_0 + Ae^{-kt}}$$

Alternative method

$T = T_0 + Ae^{-kt}$

$\frac{dT}{dt} = -kA e^{-kt}$

But from above $Ae^{-kt} = T-T_0$

$\therefore \frac{dT}{dt} = -k(T-T_0)$

(b) $T = 85^\circ$ when $t=0$.

$T = 80$ when $t=1$ mt

$T = ?$ when $t=4$ mts

$T_0 = 25^\circ$

$$T = T_0 + Ae^{-kt}$$

i.e. $T = 25 + Ae^{-kt}$

When $t=0$, $T=85$

$$85 = 25 + A$$

$$\therefore A = 60$$

When $t=1$, $T=80^0$

$\therefore\ 80 = 25 + 60 \cdot e^{-k}$

i.e. $60e^{-k} = 55$

$\therefore\ e^{-k} = \frac{55}{60}$

i.e. $-k = \log_e \frac{55}{60}$

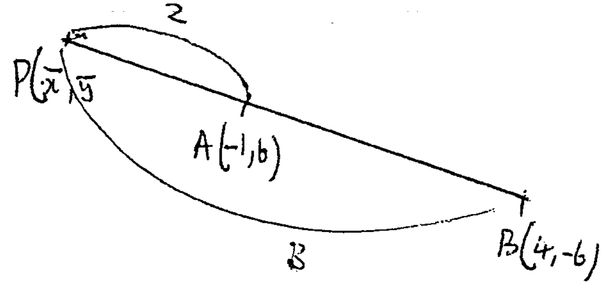
$\therefore\ k = 0.087011376$

When $t=4$ mts $T=?$

$T = 25 + 60 \cdot e^{-4k}$

$= 67.36^0$ ~~mts~~

$= 67^0$



$A\overset{x_1\ y_1}{(-1,6)}$

$B\overset{x_2\ y_2}{(4,-6)}$

$m=2$

$n=3$

$\bar{x} = \frac{mx_2 - nx_1}{m-n}$

$= \frac{2\times 4 - 3\times(-1)}{2-3}$

$= \frac{8+3}{-1}$

$= -11$

$\bar{y} = \frac{my_2 - ny_1}{m-n}$

$= \frac{2(-6) - 3\times 6}{2-3}$

$= \frac{-12-18}{-1}$

$= 30$

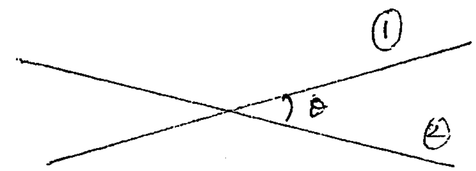
$\therefore$ P is the point $(-11, 30)$.

(e) $2x - y - 3 = 0$

$x - 3y - 7 = 0$

i.e. $y = 2x - 3$ ——— ①

$y = \frac{x}{3} - \frac{7}{3}$ ——— ②



$m_1 = 2$

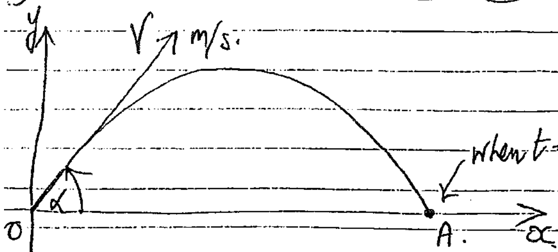
$m_2 = \frac{1}{3}$

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$

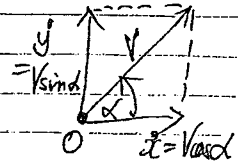
$= \frac{2 - \frac{1}{3}}{1 + 2\times\frac{1}{3}} = \frac{\frac{5}{3}}{\frac{5}{3}} = 1$

$\therefore$ The angle between the lines is 45^0

MR. SEABORN.



INITIAL VELOCITY COMPONENTS (when t=0)



$\ddot{x} = 0$

$\dot{x} = \int 0\,dt$

$= C_1$

INITIALLY when t = 0 $\dot{x} = V\cos\alpha \Rightarrow C_1 = V\cos\alpha$ and $\dot{y} = V\sin\alpha$ when $t = 0$

$\Rightarrow \dot{x} = V\cos\alpha$

$x = \int (V\cos\alpha)\,dt$

$= (V\cos\alpha)t + C_2$

When $t = 0$ $x = 0 \Rightarrow C_2 = 0$

$\Rightarrow x = (V\cos\alpha)t$ —— (1)

$\ddot{y} = -10$

$\dot{y} = \int (-10)\,dt$

$= C_3 - 10t$

$\Rightarrow C_3 = V\sin\alpha$

$\Rightarrow \dot{y} = V\sin\alpha - 10t$

$y = \int (V\sin\alpha - 10t)\,dt$

$= (V\sin\alpha)t - 5t^2 + C_4$

When $t = 0$ $y = 0 \Rightarrow C_4 = 0$

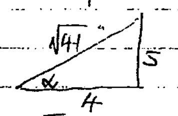
$\Rightarrow y = (V\sin\alpha)t - 5t^2$ —— (2)

At A $x = 100$, $y = 0$, $t = 5$

$\Rightarrow 100 = (V\cos\alpha)\times 5$ and $0 = (V\sin\alpha)\times 5 - 5\times 5^2$

$\Rightarrow V\cos\alpha = 20$ —— (3) $\Rightarrow V\sin\alpha = 25$ —— (4)

$\Rightarrow \frac{25}{20} = \frac{V\sin\alpha}{V\cos\alpha} \Rightarrow \tan\alpha = \frac{5}{4} \Rightarrow \alpha = \tan^{-1}\left(\frac{5}{4}\right)$



$= 51°20'$ (nearest minute).

$\Rightarrow \cos\alpha = \frac{4}{\sqrt{41}}$ and $\sin\alpha = \frac{5}{\sqrt{41}}$

Sub in (2) (or (1)) to find $V \Rightarrow V = 20 \div \frac{4}{\sqrt{41}} = 5\sqrt{41}$ m/s.

(1 MARK)

(ii) Particle is at maximum height when $\dot{y} = 0$

$\Rightarrow 0 = V\sin\alpha - 10t$

$= 25 - 10t$

$\Rightarrow t = 2.5$ s.

To find maximum height $y_{MAX} = (V\sin\alpha)t - 5t^2$ when $t = 2.5$

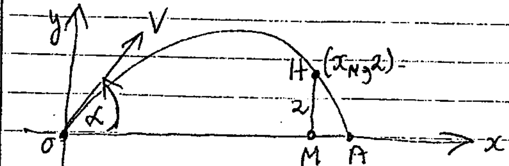
(1 MARK)

$= 25\times 2.5 - 5\times 2.5^2$

$= 31.25$ m (exactly)

To the nearest metre the maximum height is 31 m.

(b) When the man is hit (at point H on the trajectory, say)



METHOD 1

$y = (V\sin\alpha)t - 5t^2$

$2 = 25t - 5t^2$

$5t^2 - 25t + 2 = 0$

$t = \frac{25 \pm \sqrt{625 - 40}}{10}$

$= \frac{25 \pm \sqrt{585}}{10}$

$= 0.08$ s or 4.92 s (2dp)

"A few seconds later" means he is hit when $t \approx 4.92\text{s} = \frac{25 + \sqrt{585}}{10}$

$x = (V\cos\alpha)t$

$x_M = 20 \times \frac{25 + \sqrt{585}}{10}$

(3 MARKS)

$= 50 + 2\sqrt{585}$

$= 98.3735$ m (4dp)

≈ 98 m

$\Rightarrow$ The man is hit approximately 98 m from O.

METHOD 2, Equation of projectile is

$y = (\tan\alpha)x - \left(\frac{g\sec^2\alpha}{2V^2}\right)x^2$

$= \frac{5}{4} - \frac{10\left(\frac{\sqrt{41}}{4}\right)^2}{2\times(5\sqrt{41})^2} \cdot x^2$

$= \frac{5}{4} - \frac{1}{80}x^2$

$\therefore 80y = 100x - x^2$

At H: $y = 2$: $160 = 100x - x^2$

$x^2 - 100x + 160 = 0$

$\Rightarrow x = \frac{100 \pm \sqrt{10000 - 640}}{2}$

$= 50 \pm \sqrt{2340}$

$\approx 50 \pm 48.3735 \approx 98$ m since projectile hits man after a few seconds, not right away b.) $f(x) = 4x^2 - 11x + 7$

$f'(x) = 8x - 11$

$f(0.73) = 1.016$

$f'(0.73) = -5.16$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\therefore = 0.73 - \frac{1.016}{-5.16}$$

$$= 0.73 + \frac{1.016}{5.16}$$

$$= 0.9435 \text{ (4 d.p).}$$